Number sense development

in the pre-primary classroom

How is it communicated?

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In this article Paula Mildenhall describes a research project that investigated the use of a multi-semiotic teaching approach to develop flexible computational strategies in young children.

In the *Australian Curriculum: Mathematics* (Australian Curriculum Assessment and Reporting Authority [ACARA], n.d), by the end of the Foundation year, students are expected to "establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point (ACMNA001)" and "connect number names, numerals and quantities, including zero, initially up to 10 and then beyond (ACMNA002)". As the students progress into Year 1, they need to focus on representing and solving "simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts (ACMNA015)".These aspects are all components of an emerging term: number sense. Number sense has been defined by McIntosh, Reys, Reys, Bana, and Farrell (1997, p. 3) as: "A person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient ways of managing numerical situations." Crucial to the students' development of number sense is their understanding of, and being able to work flexibly with, numbers. This means that in Foundation classrooms there needs to be an emphasis on the children learning how to compare numbers and develop a sense of the magnitude of those numbers.

Due to the importance of number sense, a study was designed to explore an intentional approach to teaching the concepts. Pre-primary school children in Western Australia (Foundation year) initially engaged in a hands-on playful counting learning experience and then the children represented this newly determined number on the ten frame and the number line with the teacher focussing on certain aspects such as numerical and spatial structures, i.e., the grid pattern on the ten frame. This teaching approach of focussing on pattern and structure was informed by Mulligan's research (2010). Researchers are now emphasising the centrality in mathematics education of students using several representations in order to build up a rich understanding in mathematics (McCluskey, Mitchelmore & Mulligan, 3013; Sfard, 2008).

Using semiotic resources in the pre-primary classroom

In this article the author reports on information gathered from a larger study that involved collaboration between a researcher and a classroom teacher. This paper reports how the teacher engaged her students through the use of various signs, such as language, gesture, symbols and objects, that acted as communicative (semiotic) resources. Sfard (2008) asserts that by providing students with a rich number of realisations of a mathematical concept, students can develop

a more robust understanding of it. Sfard states that tools such as drawings, gestures and concrete materials are integral to understanding mathematics and are "the fabric of which these objects are made" (2008, p. 173).

The research

This study took place in a pre-primary classroom at a Perth school. The school has a mission to support its community, which includes many students who are Indigenous, financially disadvantaged or have English as a second language. The research entailed the teacher working collaboratively with the researcher to plan a series of nine learning activities that would engage the children and develop their number sense. It was a true collaboration with the researcher sharing her expertise as a tertiary mathematics education lecturer, including describing the ideas outlined by Sfard (2008) concerning mathematics as a 'participationist' process in which different mathematical representations play an important part. It was emphasised that the 'ten frame' is beneficial in developing place value and partitioning, and that the 'number line' supports the development of flexible number calculation. The teacher also shared her expertise, particularly by explaining the needs of her children and providing potential hands-on activities that would engage her children.

This close collaboration commenced in Term 1 and in Term 2 these nine learning activities were conducted each Friday morning with the researcher present; at the end of each learning cycle, the teacher and researcher refined the next activity in light of the children's progress (see Figure 1).

The learning activities were videoed using a high definition camera to closely capture the interactions of the teacher and students. The video footage was analysed using Erikson's iterative model (2006) where initially significant moments of teacher and student interaction were identified and these sections of video were then re-analysed to identify how the teacher and students used a range of semiotic resources. The findings from this analysis are presented in this report.

Lesson cycle 1: How many objects in the container?

The hands-on activity focussed on the pupils counting how many objects fitted into a spoon, cups and jugs. In the first week the children counted how many of larger objects fitted into the containers supplied. In the second and third week they used smaller objects and counted how many of them fitted into the containers used in the first week (which would obviously make the numbers bigger).

3 Lesson cycle Each cycle has one hands-on activity	
First lesson cycle	How many discrete objects in the container?
Second lesson cycle	How many hidden gems in the sock game?
Second lesson cycle	How many shape imprints in the play dough?
Lesson 1	• Introduce hands-on activity • Learning space/small group filmed (focus on 6 specific children) • Represent and explore number using ten frame, number line and symbols
Lesson 2	• Practice hands-on activity • Learning space/small group filmed (focus on 6 specific children) • Represent and explore number using ten frame, number line and symbols
Lesson 3	• Practice hands-on activity • Learning space/small group filmed (focus on 6 specific children) • Represent and explore number using ten frame, number line and symbols

Figure 1. The research study cycles.

The teacher set up these activities for the children to work in pairs and in this way the children received a balance of focused interactions with the teacher. With such a small group, the teacher was able to provide immediate feedback on the progress students were making. Once the children had counted the number of objects within the counters and recorded the results, the number was then represented in a several different ways (see Figure 2).

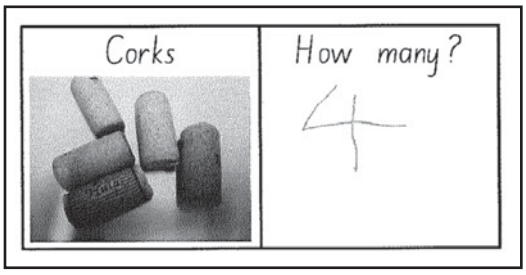

Figure 2. The children's recording of how many corks they collected.

It is noteworthy that all of the children were able to work with three different representations: the ten frame, the number line and the discrete objects in the one learning session. This was due to the teacher's expertise at engaging the children and keeping them focussed. The children represented the number created in the hands-on activity on the ten frame as a symbol and on a structured number line. The number line was very engaging due to the teacher-made sticky personalised names. The children appeared to really delight in seeing their name on the line, thus personalising this somewhat abstract representation.

The children had had previous experience in counting discrete objects and using a number line, but the ten frame was new to all of the children. The ten frame was also a new representation for the teacher, and at this stage in her teaching she did not focus on the importance of the ten frame showing a group of ten and the pattern of the 5 × 2 grid.

The teacher had a very strong physical presence when she modelled how to point at each object, which of course is essential when learning to count, and also how to organise the equipment. In Figure 3 the teacher is modelling for the child, Madie, the different aspects (the counting principles) needed to count effectively.

Figure 3. The teacher models for Madie how to count procedurally.

Typically, the teacher kept a low profile — acting as a non-participant observer and reducing her physical presence when the children were able to explore the mathematics activity independently. It was observed that the teacher was purposeful and very selective in her gesturing in Lesson cycle 1, mostly limiting the gesturing to pointing. Interestingly, the children picked up on this teacher behaviour and incorporated it into their actions.

Lesson cycle 2: Greater than/less than 5

Collaboratively, the researcher and the teacher planned the second learning activity which focussed on "more than" and "less than" 5. The teacher set up these activities so that again the children worked in pairs, playing a fun game that increased engagement. As with the previous cycle, the second cycle allowed for focussed interactions with the teacher and incidental spontaneous teaching points as the teacher observed the students' progress.

In this cycle, the teacher used the semiotic resources of the number line that had line markings, the ten frame, and a task which consisted of a game where one child hid a specified number of gems in a sock and the second child had to feel through the sock to count how many there were. This meant that the learning experience was grounded in the sense of touch. Once the number of gems had been determined, the students represented that number in different ways, as in Lesson cycle 1.

In this cycle, the children were becoming more familiar with the ten frame, although it is interesting to note that two children did not realise at the beginning of Lesson cycle 2, that when all of the counters were on the ten frame that would mean that ten items had been counted. Although the normal procedure was for the children to count how many gems were in the sock and then represent this number on the ten frame, the teacher asked an impromptu question before this process began to assess whether the children concerned were focussing on the 'ten-ness' of the ten frame.

Teacher: Remember what we played last week with the counting sock? We took and we talked about numbers that were bigger than five and numbers that were smaller than five. Have a look at the counters on your ten frame: how many have you got there? [The ten frame is full with ten counters.]
Sarah: 1, 2, 3, 4, 5, 6, 7, 8, 9. [Sarah counts out loud and emphasises the last number.]
Teacher: Is there really? Have another go.
Sarah: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. [Sarah emphasises the last number]
Teacher: Mmm [affirming]. It is called a ten frame [gestures around the ten frame] so how many do you think there are on a ten frame?
Madie: I have got ten as well!
Teacher: Really
Sarah: That is tricky.
Teacher: If it is called a ten frame how many do you think there would be on there?
Sarah and Madie:: Ten [in unison]
Teacher: So there is ten, if all the boxes are filled, you would know in your head, "Oh there must be ten."

This lack of awareness of the spatial aspects of the ten frame by the children suggests there is a need for teachers to plan purposeful activities to explore the ten frame's spatial structure. The activity in Lesson cycle 2 created the opportunity to partition numbers less than 10 as a group of 'five and some more'. The children began with a group of five on the ten frame. This was reinforced with the teacher gesturing by sweeping along the first five counters to show the line of 5 (Figure 4). This gesture was termed a "linear sweep" by the researcher. The children then partitioned the number into 5 and "how many more" (e.g., 8 = 5 + 3). Thus the children were introduced to the strategy of 'counting on'.

Figure 4. The teacher gesturing a 'linear sweep' to convey the group of five.

Lesson cycle 3: Addition of two numbers

In the final activity, the teacher also used the semiotic resources of the number line, the ten frame and an introductory task. The latter consisted of students being given a tray with play dough in which the teacher had used two different objects and created impressions. The students used tallying to record the total number of the two different imprints in the play dough tray. These were recorded on a recording sheet. The students showed this symbolically and on the ten frame. One of the shapes was shown in red on the ten frame, and the other shape was shown in yellow. The total number was then shown on the number line.

In Lesson cycle 3, the teacher set up the activity to work in pairs again. As in the previous activities, the children received some focussed interactions with the teacher.

The teacher used a sweeping linear gesture to show the children how the ten frame is

comprised of two lots of five to emphasise how the children needed to focus on the first group of five initially (developing an understanding of partitioning). In this cycle, the teacher used the ten frame to reflect the reasoning behind the 'bridging five' strategy and suggested that the children organise their counters so that they could complete such calculations as 4 + 5 = 4 + 1 + 4 (Figure 5).

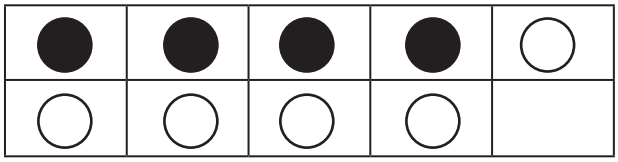

Figure 5. Using the ten frame to show the bridging five strategy.

The discourse between the teacher and children could be described as shared sustained thinking (Sylva, Melhuish, Sammons, Siraj-Blatchford & Taggart, 2004), with the teacher using questioning and thinking aloud alongside the child. The excerpt below shows this type of engagement between the teacher and with Chloe.

Chloe: Five for this one and four for this one. [Chloe points at the group.]
Teacher: Makes…?
Chloe: Nine. [Chloe does not seem to count.]
Teacher: Okay let's have a look. Now that is interesting 'cause there is only one left.
Chloe: Three, three, three. [Chloe gestures to show the counters as groups of 3.]

As the students explored the patterns on the ten frame in this third activity, the teacher also gestured a rectangular shape around the ten frame to emphasise the 'ten-ness' in the group of counters. To complete the session, the teacher also played a game of challenging the children to focus on the spaces on the ten frame rather than how many were filled up; i.e., 9 = 10 – 1. This activity was designed to lead children to develop an understanding of partitioning.

Conclusions

One important finding from this study is that the teacher's gestures appeared to have been noticed and then adopted by the children. The teacher was very selective in her gesturing and the children appeared to be very selective with their gesturing too. This suggests that pre-primary teachers need to realise the importance of employing purposeful gestures to reinforce concepts such as the ten frame. The children's use of these same gestures seems likely to reinforce their learning.

The teacher used the semiotic resources of the playful hands-on counting activity, the ten frame and the number line in conjunction with the rich combination of language, bodily movement, and gestures including pointing. This multi-semiotic teaching approach appeared to develop the children's flexible computational strategies. This suggests that pre-primary teachers should be well aware of all of the semiotic resources they use to enrich the children's learning experience when developing number sense and plan carefully how these should be introduced.

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